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Massless fermionic bound states and the gauge/gravity correspondence

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Abstract: We study the equations of motion of fermions in type IIB supergravity in the context of the gauge/gravity correspondence. The main motivation is the search for normalizable fermionic zero modes in such backgrounds, to be interpreted as composite massless fermions in the dual theory. We specialize to backgrounds characterized by a constant dilaton and a self-dual three-form. In the specific case of the Klebanov-Strassler solution we construct explicitly the fermionic superpartner of the Goldstone mode associated with the broken baryonic symmetry. The fermionic equations could also be used to search for goldstinos in theories that break supersymmetry dynamically.

Keywords: Gauge-gravity correspondence, Supergravity Models, Supersymmetry Breaking.

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1. Introduction and summary

One of the latest surges of interest in the context of the gauge/gravity correspondence (for reviews close to the topics of this work, see [1−3]) has been the possibility that some backgrounds might provide the supergravity realization of dynamical supersymmetry (SUSY) breaking. This possibility was first considered for the quiver theories described in [4−6]. These theories were constructed as a non-conformal deformation of the conformal theories [7−9] dual to the new Sasaki-Einstein manifolds [10, 11] Y^{pq}. Unfortunately, in spite of being chiral theories, these theories do not display true dynamical supersymmetry breaking with a stable ground state, but rather a runaway behavior [5, 12] very much like super QCD with $0 < N_f < N_c$ [13]. Still, the possibility of the existence of gravity solutions dual to dynamically broken SUSY has not been ruled out. (Some work on deformations for these theories can be found in [14−18]. For related earlier work see [19].)

One of the consequences of spontaneous SUSY breaking (dynamical [20] or tree level [21, 22]) is the existence of a fermionic Goldstone mode g — the "goldstino" [23]. Such mode can arise as a massless bound state of microscopic degrees of freedom in a confining theory and has the distinguishing property of coupling to the supercurrent J without derivative terms — in obvious notation:

$$\langle 0|J^{\mu}_{\alpha}|g_{\beta}\rangle = f\gamma^{\mu}_{\alpha\beta}, \tag{1.1}$$

where $f \neq 0$ is the goldstino coupling. In the context of the gauge/gravity correspondence, such particle must be described by a normalizable zero mode in the bulk coupling directly to the gravitino Ψ_μ and can be studied by looking at the bulk fermionic equations of motion.

Even in theories that do not break SUSY, the study of the bulk fermionic equations and the search for normalizable zero modes is still of interest. Obviously, with unbroken SUSY, the bosonic and fermionic spectra must match and one does not obtain additional information from the latter. However, particularly in the case of zero modes, some information may be easier to obtain in the second case, since the fermionic equations are easily linearized and index theorems may be available. In some cases, such as the cascading theory of Klebanov and Strassler (KS) [24] massless fermionic modes (the "axino")[1] must exist as a superpartner to the Goldstone boson associated with the breaking of the baryonic U(1) symmetry [25 – 27] and their explicit construction strengthens the correspondence. More generically, $\mathcal{N} = 1$ SUSY implies the presence of massless fermionic superpartners of the scalar fields parameterizing the quantum moduli space of vacua, when there is one (some of these scalars can be seen as Goldstone bosons of broken global continuous symmetries).

The "axino" does not obey (1.1) (since SUSY is unbroken in this case) and its explicit form helps elucidating precisely how (1.1) should be interpreted in the bulk. The solution that we find in section 4 has the property that it does not give a source for the supercovariant field strength of the gravitino, more specifically

$$\Gamma^{MNP}\mathcal{D}_N\Psi_P = 0 \qquad \text{on shell.} \tag{1.2}$$

(The notation is discussed in section 2). We propose that the signature of spontaneous SUSY breaking is the existence of a normalizable zero mode for which (1.2) is not satisfied.

Another way to distinguish a generic massless fermion from the goldstino is by looking at how they transform under the global symmetries of the problem.[2] For instance, the bosonic zero modes found in [26] are odd under the Z_2 symmetry exchanging the two S^2 spheres of the deformed conifold and the same symmetry should act non-trivially on their fermionic superpartner. On the other hand, a true goldstino should be invariant under such symmetries. We will discuss the details for the KS solution in the conclusions after having presented the explicit solution.

The purpose of this paper is twofold — on the one hand, we wish to begin addressing the general issues above for a class of KS-like backgrounds (consisting of a constant dilaton and a self-dual three-form) and, on the other, we test these techniques in the true KS model [24] and construct explicitly the fermionic zero mode. Eventually, one will have to consider more complicated backgrounds with more general fluxes but we feel that the class we are considering in this paper is a good starting point to sharpen one's tools and includes at least the important example of [24]. Various aspects of flux compactifications that might be relevant in this context are reviewed in [28].

Perhaps the most interesting quality of the general equations we discuss is that the existence of a zero mode hinges on the existence of a solution to the massless Dirac equation on a (six-dimensional) Ricci flat manifold (see section 3.4). In the compact case, the existence of such a solution implies the existence of a covariantly constant spinor and

[1]Strictly speaking, one should refrain to call such multiplet "axionic" since it is not related to an anomalous symmetry, like the QCD axion. Still, we will, in a few places, use the word within quotes for sake of brevity and to make connection with the previous literature.

[2]We thank I. Klebanov for pointing out this possibility to us in the context of the KS solution.

thus of a Kähler structure, by the standard arguments of integration by part. In the non-compact case however, the boundary terms cannot be neglected and, because of the presence of the warp factor, there is a possibility for having a normalizable zero mode without necessarily implying a Kähler structure. We shall discuss this possibility in the conclusions, after having presented the dependence of the equations from the warp factor. Another possibility would be to leave the Kähler structure untouched but change the three-form appropriately.

The paper is organized as follows: In section 2 we begin by reviewing the fermionic equations of motion of type IIB supergravity obtained in [29] (see also [30−33]). In section 3 we specialize to the above mentioned class of backgrounds and show how the equations for the zero modes can be reduced to a set of Dirac and Rarita-Schwinger equations on the internal manifold, starting precisely with the Dirac equation discussed above. In section 4 we turn to an application of the equations just derived and use them to construct the "axino" for the true KS solution. This zero mode is the fermionic partner of the Goldstone mode associated to the breaking of the baryonic U(1) symmetry and is not to be thought as a goldstino and in fact condition (1.2) is satisfied. We briefly summarize our findings in section 5 and present a more detailed discussion of the Z_2 symmetry transformations of the KS solution and comment on the issue related to the Kähler structure mentioned above. Some useful formulas, like the explicit expression for the spin connection on the deformed conifold, are collected in the appendix.

2. The fermionic equations of motion of type IIB supergravity

In this section we review the fermionic equations of motion of type IIB supergravity obtained in [29]. This allows us to make some comments on the conventions and notation used. We will set the Newton constant to one, $\kappa = 1$, for convenience. (It can always be reinstated by dimensional analysis.) We will only work to first order in the fermionic fields.

In order to follow the more recent literature, we will use, contrary to [29], a "mostly plus" metric. This can be most easily accomplished by letting $g_{MN} \to -g_{MN}$, $\Gamma_M \to i\Gamma_M$ and so on, and implies a few sign changes that are easily implemented. The Γ-matrices are all real in the Majorana representation.

Our convention for the ϵ-tensor is that it includes the appropriate determinant of the metric and thus transforms as a true tensor, not as a density. Also, when evaluated with flat indices it is purely numerical and we have the sign convention $\epsilon_{0\ldots9} = -\epsilon^{0\ldots9} = 1$. Finally, the five-form F_5 is self-dual ($*_{10}F_5 = F_5$) in the sense

$$F_{M_1M_2M_3M_4M_5} = \frac{1}{5!}\epsilon_{M_1M_2M_3M_4M_5M_6M_7M_8M_9M_{10}}F^{M_6M_7M_8M_9M_{10}} \tag{2.1}$$

We also define, with flat indices, $\Gamma_{\chi 10} = \Gamma^0 \ldots \Gamma^9$, and the chiralities of the dilatino λ and gravitino Ψ_M are: $\Gamma_{\chi 10}\lambda = -\lambda$, $\Gamma_{\chi 10}\Psi_M = +\Psi_M$, reversed from the conventions in [29].

The dilatino and gravitino equations of motion are, respectively [29]:

$$\Gamma^M D_M\lambda = \frac{i}{240}\Gamma^{MNPQR}F_{MNPQR}\lambda + \frac{1}{24}\Gamma^M\Gamma^{NPQ}G_{NPQ}\Psi_M \\ +\Gamma^M\Gamma^R P_R\Psi^*_M, \tag{2.2}$$

and

$$\Gamma^{MNP}D_N\Psi_P = -\frac{1}{48}\Gamma^{NRL}\Gamma^M G^*_{NRL}\lambda - \frac{i}{480}\Gamma^{MNP}\Gamma^{QRLST}F_{QRLST}\Gamma_N\Psi_P \\ +\frac{1}{96}\Gamma^{MNP}(\Gamma_N{}^{LSR}G_{LSR} - 9\Gamma^{LS}G_{NLS})\Psi^*_P + \frac{1}{2}\Gamma^R\Gamma^M P_R\lambda^*. \tag{2.3}$$

Note that the Ψ terms on the r.h.s. of (2.2) and (2.3) are not written out explicitly in [29] but they are certainly present for supercovariance as can be seen by taking the SUSY variations, which in our notation read:

$$\delta\lambda = \Gamma^M P_M\varepsilon^* + \frac{1}{24}\Gamma^{MNP}G_{MNP}\varepsilon, \tag{2.4}$$

and

$$\delta\Psi_M = D_M\varepsilon + \frac{i}{480}\Gamma^{NPQRS}F_{NPQRS}\Gamma_M\varepsilon - \frac{1}{96}(\Gamma_M{}^{NPQ}G_{NPQ} - 9\Gamma^{NP}G_{MNP})\varepsilon^*. \tag{2.5}$$

We have chosen to write out explicitly all the fermionic terms to avoid confusion but one could just as well introduce a supercovariant derivative $\mathcal{D}_N$ in terms of which eq. (2.3) becomes simply

$$\Gamma^{MNP}\mathcal{D}_N\Psi_P = -\frac{1}{48}\Gamma^{NRL}\Gamma^M G^*_{NRL}\lambda + \frac{1}{2}\Gamma^R\Gamma^M P_R\lambda^*. \tag{2.6}$$

The r.h.s. of (2.6) acts as a source for the supercovariant field strength of the gravitino.

The ordinary covariant derivatives are by definition:

$$D_M\lambda = \left(\partial_M + \frac{1}{4}\omega^{AB}_M\Gamma_{AB} - \frac{3}{2}iQ_M\right)\lambda \tag{2.7}$$

$$D_M\Psi_R = \left(\partial_M + \frac{1}{4}\omega^{AB}_M\Gamma_{AB} - \frac{1}{2}iQ_M\right)\Psi_R - \mathbf{\Gamma}^L_{MR}\Psi_L, \tag{2.8}$$

where Q_M is the auxiliary U(1) field introduced in [29] and ω^{AB}_M the usual spin connection. Notice that the contribution of the Christoffel symbol $\mathbf{\Gamma}^L_{MR}$ drops out in the kinetic term for the gravitino but, without it, the derivative is no longer covariant.

3. The fermionic equations of motion in a KS-like ansatz

We now specialize the equations reviewed in the previous section to a generic KS-like background precisely defined as follows.

3.1 Bosonic ansatz

Let us review the ansatz step by step in order to distinguish between the basic assumptions and their consequences. We start from the 4+6 split of the geometry. The ten dimensional metric is split into a four-dimensional warped Minkowski space described by the coordinates x^μ and a six-dimensional internal space described by the coordinates y^i:

$$ds_{10}^2 = e^{-\frac{1}{2}u(y)} dx^\mu dx_\mu + e^{\frac{1}{2}u(y)} d\hat{s}^2 \tag{3.1}$$

where $e^{u(y)}$ is the warp factor and $d\hat{s}^2 = g_{ij}(y) dy^i dy^j$ is the internal metric which is assumed to describe a smooth non-compact manifold.

To avoid confusion we stress that the six-dimensional indices $i, j \dots$ will always be raised/lowered with the metric g_{ij} and all powers of the warp factor written explicitly. Also, with a slight abuse of notation, the covariant derivative D_i will denote the true covariant derivative on the internal manifold, and thus it is shifted (by a term containing the warp factor) with respect to the one used in section 2. A subtlety that arises when commuting it through a Γ-matrix is discussed in appendix A.

By Poincaré invariance in the Minkowski space, all other fields can depend only on the y^i coordinates. Furthermore, the complex 3-form G_3 must be living purely in the six-dimensional internal space.

The basic assumption that we make is to take the 3-form to be imaginary self-dual in the six-dimensional internal space:

$$*_6 G_3 = i G_3, \tag{3.2}$$

that is

$$\frac{1}{6} \epsilon_{ijklmn} G^{lmn} = i G_{ijk}, \tag{3.3}$$

where the ϵ-tensor is defined with respect to the internal metric g_{ij}. In particular, for flat indices we have $\epsilon_{4\dots 9} = \epsilon^{4\dots 9} = 1$.

The assumption (3.2) leads to many simplifications. First of all, we can consider a background where the type IIB dilaton and RR scalar can be held constant, thus allowing us to set

$$P_M = Q_M = 0. \tag{3.4}$$

We can think of this condition as a kind of extremality condition, since the equations of motion for the dilaton and axion are sourceless for our ansatz. The Bianchi identities further impose that the 3-form is closed:

$$dG_3 = 0, \tag{3.5}$$

and self-duality thus requires it to be harmonic.

To preserve 4d Poincaré symmetry the self-dual 5-form must be taken as:

$$F_5 = \mathcal{F}_5 + *_{10}\mathcal{F}_5, \qquad \mathcal{F}_5 = \mathcal{F}_1 \wedge dx^0 \wedge dx^1 \wedge dx^2 \wedge dx^3. \tag{3.6}$$

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The equations of motion of the 5-form are:

$$dF_5 = \frac{1}{8} i G_3 \wedge G_3^*. \tag{3.7}$$

Since G_3 is purely in the 6-manifold, the EOM above imply that $d\mathcal{F}_5 = 0$ and thus $\mathcal{F}_1 = dZ$, with $Z = Z(y)$ a real function.

Now the EOM for the 3-form are:

$$d *_{10} G_3 = 4i F_5 \wedge G_3. \tag{3.8}$$

Taking into account self-duality of G_3, we have:

$$*_{10} G_3 = i e^{-u} G_3 \wedge dx^0 \wedge dx^1 \wedge dx^2 \wedge dx^3. \tag{3.9}$$

Thus the EOM for G_3 imply that $d(4Z - e^{-u}) \wedge G_3 = 0$ over the 6-manifold, which, due to self-duality of G_3 implies that $Z = \frac{1}{4} e^{-u}$ up to an additive constant that we set to zero. Thus:

$$\mathcal{F}_5 = \frac{1}{4} de^{-u} \wedge dx^0 \wedge dx^1 \wedge dx^2 \wedge dx^3. \tag{3.10}$$

Note that the sign of the 5-form is directly related to the sign in the self-duality equation for G_3.

The Einstein equations for the metric, given the above source fields, yield for the internal part just the condition that the 6-dimensional metric is Ricci-flat, $R_{ij} = 0$. For the 4-dimensional part, they yield an equation for the warp factor that can be also consistently derived from the EOM of the 5-form (3.7) with indices along the 6-manifold, and which is entirely determined by the data on the six-dimensional manifold:

$$-\nabla_6 e^u = \frac{1}{12} G^*_{lmn} G^{lmn} \tag{3.11}$$

Of course, the above equations do not imply SUSY. As well known [34], SUSY requires in addition the internal space to be Kähler and the three-form to be $(2,1)$ and primitive. The KS background obeys these conditions and is thus supersymmetric. However, we will not make this assumption in our derivation, except in section 4 where we shall specialize to the KS background.

Without (or even with) SUSY, one might wonder whether it makes sense to impose the self-duality condition on the 3-form. Relaxing this condition would imply a much more generic, but also much more complicated, set up. Though such a generalization should ultimately be carried out, we feel that the above set up is first of all a good training ground, but might also be of relevance in situations in which SUSY is present asymptotically, and the 3-form could well preserve its self-duality everywhere.

3.2 Fermionic ansatz

Now it is time to introduce a 4-6 split for the spinors and the Γ-matrices. We split the Γ-matrices as follows

$$\Gamma^\mu = e^{\frac{u}{4}} \gamma^\mu \otimes \mathbf{1}, \quad \Gamma^i = e^{-\frac{u}{4}} \gamma_{\chi 4} \otimes \gamma^i, \quad \text{with} \quad \gamma_{\chi 4} = i\gamma^0 \ldots \gamma^3. \tag{3.12}$$

The warp factors have been denoted explicitly so that the four and six dimensional γ-matrices obey

$$\{\gamma^\mu, \gamma^\nu\} = 2\eta^{\mu\nu} \quad \text{and} \quad \{\gamma^i, \gamma^j\} = 2g^{ij}. \tag{3.13}$$

We are in a Majorana representation where all γ^μ are real and all γ^i imaginary. Similar equations, with the warp factors reversed, hold for Γ_μ and Γ_i. We also define, with flat indices,

$$\gamma_{\chi 6} = -i\gamma^4 \dots \gamma^9, \tag{3.14}$$

which is such that $\Gamma_{\chi 10} = \gamma_{\chi 4} \otimes \gamma_{\chi 6}$.

We consider one of the two linearly independent constant Weyl spinors in four dimensions ϵ_+ of positive four dimensional chirality together with its complex conjugate $\epsilon_- = \epsilon_+^*$ of negative four dimensional chirality. We make the most general ansatz that is suited to search for zero momentum massless modes with four-dimensional spin 1/2:

$$\begin{aligned} \lambda &= \epsilon_+ \otimes \lambda_- + \epsilon_- \otimes \lambda_+ \\ \Psi_\mu &= \Gamma_\mu(\epsilon_+ \otimes \chi_- + \epsilon_- \otimes \chi_+) \\ \Psi_i &= e^{\frac{u}{4}}(\epsilon_+ \otimes \psi_{+i} + \epsilon_- \otimes \psi_{-i}). \end{aligned} \tag{3.15}$$

The $\pm$ signs denote the four and six dimensional chiralities and in the case of λ we use the same symbol for the six dimensional spinor as for the ten dimensional one since no confusion can arise. The warp factor in the last of (3.15) has been introduced for convenience. Notice that, apart from $\epsilon_\pm$, the other spinors are not the complex conjugate of each other since the ten dimensional fermions are not Majorana.

3.3 Fermionic equations of motion, preliminaries

It is now straightforward to insert (3.15) and the bosonic ansatz into the equations of motion (2.2), (2.3) and to collect the terms proportional to ϵ_+ and those proportional to ϵ_-. We obtain equations that contain only data from the six dimensional manifold. Namely, the dilatino equation (2.2) gives rise to the following two equations:

$$\gamma^i D_i \lambda_- + \frac{3}{8}\gamma^i \partial_i u \lambda_- = \frac{1}{4} e^{-\frac{u}{2}} \gamma_{jk} G^{ijk} \psi_{+i} \tag{3.16}$$

and:

$$\gamma^i D_i \lambda_+ - \frac{1}{8}\gamma^i \partial_i u \lambda_+ = -\frac{1}{6} e^{-\frac{u}{2}} \gamma_{ijk} G^{ijk} \chi_+. \tag{3.17}$$

Similarly, the component along x^μ of (2.3) gives rise to:

$$\begin{aligned} &\gamma^{ij} D_i \psi_{+j} - \frac{1}{2}\partial^i u \psi_{+i} + \frac{3}{8}\gamma^{ij}\partial_i u \psi_{+j} + 3\gamma^i D_i \chi_- - \frac{3}{8}\gamma^i \partial_i u \chi_- \\ &= \frac{1}{48} e^{-\frac{u}{2}} \gamma^{ijk} G^*_{ijk} \lambda_- + \frac{1}{8} e^{-\frac{u}{2}} G^{nij} \gamma_{ij} \psi^*_{-n} \end{aligned} \tag{3.18}$$

and:

$$\begin{aligned} &\gamma^{ij} D_i \psi_{-j} + \frac{3}{8}\gamma^{ij}\partial_i u \psi_{-j} - 3\gamma^i D_i \chi_+ - \frac{9}{8}\gamma^i \partial_i u \chi_+ \\ &= \frac{1}{8} e^{-\frac{u}{2}} \gamma^{ijk} G_{ijk} \chi^*_- - \frac{1}{8} e^{-\frac{u}{2}} G^{nij} \gamma_{ij} \psi^*_{+n} \end{aligned} \tag{3.19}$$

Finally, the component along y^i of (2.3) yields:

$$\gamma^{pij}D_i\psi_{+j} - \frac{1}{8}\gamma^{pij}\partial_i u\psi_{+j} + 4\gamma^{pi}D_i\chi_- - \frac{1}{2}\gamma^{pi}\partial_i u\chi_- = \frac{1}{2}e^{-\frac{u}{2}}G^{pij}\gamma_{ij}\chi_+^* \tag{3.20}$$

and

$$\begin{aligned}&\gamma^{pij}D_i\psi_{-j} + \frac{3}{8}\gamma^{pij}\partial_i u\psi_{-j} - 4\gamma^{pi}D_i\chi_+ + \frac{1}{2}\gamma^{pi}\partial_i u\chi_+ - 2\partial^p u\chi_+ \\ &= \frac{1}{8}e^{-\frac{u}{2}}G^{*pij}\gamma_{ij}\lambda_+ + \frac{1}{2}e^{-\frac{u}{2}}G^{pij}\gamma_{ij}\chi_-^* - \frac{1}{2}e^{-\frac{u}{2}}G^{pij}\gamma_i\psi_{+j}^*\end{aligned} \tag{3.21}$$

Before making any further manipulation, it is advisable to check which of the six dimensional fermions can or cannot be gauged away in this particular bosonic background. Therefore we reserve to the SUSY variations (2.4) and (2.5) the same treatment we gave the equations of motion. For the SUSY variation:

$$\varepsilon = \epsilon_+ \otimes \varepsilon_+ + \epsilon_- \otimes \varepsilon_- : \tag{3.22}$$

(where, again, ε_+ and ε_- are independent), we get:

$$\begin{aligned}\delta\lambda_+ &= 0 \\ \delta\lambda_- &= \frac{1}{24}e^{-\frac{3u}{4}}G_{ijk}\gamma^{ijk}\varepsilon_+ \\ \delta\chi_+ &= 0 \\ \delta\chi_- &= -\frac{1}{4}e^{-\frac{u}{4}}\gamma^i\partial_i u\varepsilon_+ - \frac{1}{96}e^{-\frac{3u}{4}}G_{ijk}\gamma^{ijk}\varepsilon_-^* \\ e^{\frac{u}{4}}\delta\psi_{+i} &= D_i\varepsilon_+ + \frac{1}{4}\gamma_{ij}\partial^j u\varepsilon_+ - \frac{1}{8}\partial_i u\varepsilon_+ + \frac{1}{16}e^{-\frac{u}{2}}G_{ijk}\gamma^{jk}\varepsilon_-^* \\ e^{\frac{u}{4}}\delta\psi_{-i} &= D_i\varepsilon_- + \frac{1}{8}\partial_i u\varepsilon_- + \frac{1}{8}e^{-\frac{u}{2}}G_{ijk}\gamma^{jk}\varepsilon_+^*\end{aligned} \tag{3.23}$$

The usual gauge choice $\Gamma^M\psi_M = 0$ can be easily seen to correspond to $4\chi_- + \gamma^i\psi_{+i} = 0$ and $4\chi_+ - \gamma^i\psi_{-i} = 0$, but we will choose a more convenient one in the following.

3.4 Disentangling the fermionic equations of motion

We now rewrite all the fermionic equations as a system which can be solved step by step, in principle by inverting the Dirac operator on the 6 dimensional transverse manifold.

First of all, we subtract from (3.19) the contraction with γ_p of (3.21), to obtain the massless Dirac equation discussed in the introduction:

$$\gamma^i D_i\tilde{\chi}_+ = 0, \qquad \text{where} \qquad \tilde{\chi}_+ = e^{-\frac{5}{8}u}\chi_+. \tag{3.24}$$

In order to rewrite (3.20), we define

$$\tilde{\psi}_{+i} = e^{-\frac{u}{8}}(\psi_{+i} + \gamma_i\chi_-). \tag{3.25}$$

If we then choose the gauge $\gamma^i\tilde{\psi}_{+i} = 0$, we obtain the following simple equation:

$$\gamma^j D_j\tilde{\psi}_{+i} = \frac{1}{2}G_{ijk}\gamma^{jk}\tilde{\chi}_+^*. \tag{3.26}$$

Note that contracting with γ^i we obtain the condition (that was used to obtain the previous equation):

$$D^i\tilde{\psi}_{+i} = 0. \tag{3.27}$$

Now we turn to (3.16)–(3.17). We perform the rescalings:

$$\tilde{\lambda}_+ = e^{-\frac{u}{8}}\lambda_+, \qquad \tilde{\lambda}_- = e^{\frac{3}{8}u}\lambda_-. \tag{3.28}$$

Then the equations simply write:

$$\gamma^i D_i\tilde{\lambda}_+ = -\frac{1}{6}G_{ijk}\gamma^{ijk}\tilde{\chi}_+, \tag{3.29}$$

$$\gamma^i D_i\tilde{\lambda}_- = \frac{1}{4}G^{ijk}\gamma_{jk}\tilde{\psi}_{+i}. \tag{3.30}$$

Turning to (3.21), we define:

$$\tilde{\psi}_{-i} = e^{\frac{3}{8}u}\psi_{-i}. \tag{3.31}$$

Then, imposing the gauge $\gamma^i\tilde{\psi}_{-i} = 0$, we obtain the equation:

$$\gamma^j D_j\tilde{\psi}_{-i} = -4e^u D_i\tilde{\chi}_+ - \gamma_{ij}\partial^j e^u\tilde{\chi}_+ - \partial_i e^u\tilde{\chi}_+ + \frac{1}{8}G^*_{ijk}\gamma^{jk}\tilde{\lambda}_+ - \frac{1}{2}G_{ijk}\gamma^j\tilde{\psi}^{*k}_+. \tag{3.32}$$

For the sake of completeness, the contraction with γ^i gives:

$$D^i\tilde{\psi}_{-i} = -3\gamma^i\partial_i e^u\tilde{\chi}_+. \tag{3.33}$$

We are left with (3.18). After we perform an additional rescaling:

$$\tilde{\chi}_- = e^{\frac{7}{8}u}\chi_-, \tag{3.34}$$

the equation becomes:

$$\gamma^i D_i\tilde{\chi}_- = -\frac{1}{2}\partial^i e^u\tilde{\psi}_{+i} - \frac{1}{96}G^*_{ijk}\gamma^{ijk}\tilde{\lambda}_- - \frac{1}{16}G^{ijk}\gamma_{ij}\tilde{\psi}^*_{-k}. \tag{3.35}$$

Note that the SUSY variation of the gauge fixing conditions is simply given by:

$$\gamma^i\delta\tilde{\psi}_{+i} = \gamma^i D_i\tilde{\varepsilon}_+, \qquad \varepsilon_+ = e^{\frac{3}{8}u}\tilde{\varepsilon}_+, \tag{3.36}$$

$$\gamma^i\delta\tilde{\psi}_{-i} = \gamma^i D_i\tilde{\varepsilon}_-, \qquad \varepsilon_- = e^{-\frac{u}{8}}\tilde{\varepsilon}_-. \tag{3.37}$$

4. Finding an explicit fermionic solution in the KS background

In this section, we apply the equations derived above to study the problem of finding a fermionic massless zero mode in the supersymmetric KS background [24]. The existence of such mode is needed in order to form a SUSY multiplet together with the two bosonic massless modes (sometimes referred to as the "axion" and the "saxion" with a slight abuse of language) which have been derived in [26, 27].[3] The "axion" is actually the Goldstone

[3] After the bosonic modes were given, the full baryonic branch was constructed in [35], using the techniques of [36] and showed to agree with the gauge theory analysis in [37]. Another non-SUSY deformation was considered in [38].

boson associated with the breaking of the baryonic symmetry [25 – 27]. Finding the "axino" completes the holographic description of the massless multiplet present in the low energy effective description of the boundary theory, and is thus a nice check of the gauge/gravity correspondence.

We first review the KS background [24]. This is just a specific case of the generic ansatz discussed in section 3.1 and thus, we only need to know that the internal space is the deformed conifold [39] (see also [40 – 42]), whose sechsbein are, up to an overall rescaling:

$$\begin{aligned}
e^1 &= A(\tau)(-\sin\theta_1 d\phi_1 - \cos\psi\sin\theta_2 d\phi_2 + \sin\psi d\theta_2) \\
e^2 &= A(\tau)(d\theta_1 - \sin\psi\sin\theta_2 d\phi_2 - \cos\psi d\theta_2) \\
e^3 &= B(\tau)(-\sin\theta_1 d\phi_1 + \cos\psi\sin\theta_2 d\phi_2 - \sin\psi d\theta_2) \\
e^4 &= B(\tau)(d\theta_1 + \sin\psi\sin\theta_2 d\phi_2 + \cos\psi d\theta_2) \\
e^5 &= C(\tau)(d\psi + \cos\theta_1 d\phi_1 + \cos\theta_2 d\phi_2) \\
e^6 &= C(\tau)d\tau
\end{aligned} \tag{4.1}$$

where, in terms of the function:

$$K(\tau) = \frac{(\sinh\tau\cosh\tau - \tau)^{1/3}}{\sinh\tau}, \tag{4.2}$$

defined in [24], we have:

$$\begin{aligned}
A^2(\tau) &= \frac{1}{4}K(\tau)(\cosh\tau - 1), \\
B^2(\tau) &= \frac{1}{4}K(\tau)(\cosh\tau + 1), \\
C^2(\tau) &= \frac{1}{3K^2(\tau)}.
\end{aligned} \tag{4.3}$$

For the sake of completeness, we give the spin connection in appendix A. We can then write the equations for a covariantly constant spinor on the deformed conifold, $D_i\eta = 0$. With our coordinate choice (4.1), they imply that the spinor must be constant, and has to obey the conditions (with flat indices):

$$(\gamma^{12} + \gamma^{34})\eta = (\gamma^{16} - \gamma^{45})\eta = 0 \tag{4.4}$$

We will choose η to have positive chirality $\gamma_{\chi^6}\eta = \eta$ and denote its complex conjugate (of negative chirality) by η^*. Then, η satisfies three conditions:

$$(\gamma^1 + i\gamma^4)\eta = 0, \qquad (\gamma^3 + i\gamma^2)\eta = 0, \qquad (\gamma^6 - i\gamma^5)\eta = 0. \tag{4.5}$$

The above formula allows one to read off the complex structure in the flat indices. Denoting complex indices in boldface, we take the following holomorphic sechsbein:

$$e^{\mathbf{1}} = e^1 + ie^4, \qquad e^{\mathbf{2}} = e^3 + ie^2, \qquad e^{\mathbf{3}} = e^6 - ie^5, \tag{4.6}$$

and the antiholomorphic sechsbein are obtained by complex conjugation. The unconventional combinations above are forced upon us by the labeling of the sechsbein (4.1) that is the one commonly used in the literature. With these normalizations, the flat metric is $\eta^{\mathbf{1}\bar{\mathbf{1}}} = 2$ and $\eta_{\mathbf{1}\bar{\mathbf{1}}} = 1/2$.

With (4.6), the conditions (4.5) simply become:

$$\gamma^{\mathbf{a}}\eta = 0, \qquad \mathbf{a} = \mathbf{1}, \mathbf{2}, \mathbf{3}. \tag{4.7}$$

Using the covariantly constant spinor η, we can now start solving the fermionic equations in the KS background.

Eq. (3.24) can be trivially solved by setting:

$$\tilde{\chi}_+ = \eta. \tag{4.8}$$

Notice that the expression for $\chi_+ = e^{\frac{5}{8}u}\eta$ is then normalizable in the sense of [43] (see also [44 − 46], the case for the Rarita-Schwinger field is discussed in [47 − 50]).

We already notice from the asymptotic behavior of the solution above, that the mode we have found should be dual to an operator of dimension $\Delta = \frac{5}{2}$, which is the right dimension for the fermion in the "axion-saxion" chiral multiplet, which has dimension $\Delta = 2$, see [26].

Moving on to (3.26), we need, first of all, an expression for the three form G_3. This is given in [24] and has the form (in the flat basis (4.6)):

$$\begin{aligned} G_3 = \sqrt{3}M \Bigg[&\frac{(\tau\cosh\tau - \sinh\tau)}{\sinh^3\tau} e^{\mathbf{1}} \wedge e^{\mathbf{2}} \wedge e^{\bar{\mathbf{3}}} \\ &+ \frac{(\sinh\tau\cosh\tau - \tau)}{2\sinh^3\tau}(e^{\bar{\mathbf{1}}} \wedge e^{\mathbf{2}} \wedge e^{\mathbf{3}} - e^{\mathbf{1}} \wedge e^{\bar{\mathbf{2}}} \wedge e^{\mathbf{3}})\Bigg], \end{aligned} \tag{4.9}$$

where M is the number of fractional branes in the KS set up. In relation to the complex structure (4.6) G_3 is indeed a $(2,1)$ primitive form, so that $G_{ijk}\gamma^{ijk}\eta = 0$.

It can also be easily checked that the r.h.s. of (3.26) has only antiholomorphic indices and thus it is consistent[4] to take, in the flat basis:

$$\tilde{\psi}_{+\mathbf{a}} = 0. \tag{4.10}$$

Making now the ansatz that $\tilde{\psi}_{+i}$ depends explicitly only on τ one can show (by requiring the θ_1 dependence of (3.26) to cancel out algebraically) that the most general form for the remaining components is:

$$\begin{aligned} \tilde{\psi}_{+\bar{\mathbf{1}}} &= z\gamma^{\mathbf{1}}\eta^* + v\gamma^{\mathbf{2}}\eta^* \\ \tilde{\psi}_{+\bar{\mathbf{2}}} &= v\gamma^{\mathbf{1}}\eta^* + z\gamma^{\mathbf{2}}\eta^* \\ \tilde{\psi}_{+\bar{\mathbf{3}}} &= -2z\gamma^{\mathbf{3}}\eta^* \end{aligned} \tag{4.11}$$

[4]Indeed, the spin connection is such that the covariant derivative does not couple holomorphic and antiholomorphic indices (with respect to the flat basis (4.6)).

The terms proportional to $z(\tau)$ are solutions of an homogeneous equation whereas $v(\tau)$ couples to the source. The remaining conditions are all solved by the functions

$$\begin{aligned} z(\tau) &= \frac{c}{\sinh\tau\cosh\tau - \tau} \\ v(\tau) &= -M\frac{(\tau\cosh\tau - \sinh\tau)}{K\sinh^2\tau}. \end{aligned} \tag{4.12}$$

Requiring $\tilde{\psi}_{+i}$ to be regular at the origin sets $c = 0$.[5] Normalizability can be checked using the boundary terms discussed in [47 – 50].

Armed with the explicit solutions of (3.24) and (3.26) we can easily solve the dilatino equations (3.29) and (3.30). The source for (3.29) is identically zero and normalizability forces us to take

$$\lambda_+ = 0 \tag{4.13}$$

The source for (3.30) turns out to be proportional to $\gamma^{\mathbf{3}}\eta^*$ times an overall function of τ allowing for the ansatz $\tilde{\lambda}_- = f(\tau)\eta^*$. Inserting in (3.30) we find $f = 4e^u$, implying the normalizability of

$$\lambda_- = 4e^{\frac{5}{8}u}\eta^* = 4\chi_+^*. \tag{4.14}$$

Notice that any dependence on λ_- disappears in the remaining equations.

It is now time to look at (3.32). It will be useful to have the explicit expression for the warp factor. With the normalizations (4.1) and (4.9), we have, from (3.11):

$$e^{u(\tau)} = 2M^2\int_\tau^\infty d\tau' K(\tau')\frac{(\tau'\coth\tau' - 1)}{\sinh\tau'}. \tag{4.15}$$

In this case the source term has both holomorphic and antiholomorphic indices and the resulting set of equations cannot be solved in terms of elementary functions. Still, it is possible to completely characterize the solution and its asymptotic behavior in terms of the warp factor. Making the ansatz that $\tilde{\psi}_{-i}$ only depends explicitly on τ and imposing the gauge condition, one can write $\tilde{\psi}_{-i}$ in terms of three unknown functions:

$$\begin{aligned} &\tilde{\psi}_{-\mathbf{1}} = r(\tau)\gamma^{\bar{\mathbf{1}}}\eta, \qquad \tilde{\psi}_{-\mathbf{2}} = r(\tau)\gamma^{\bar{\mathbf{2}}}\eta, \qquad \tilde{\psi}_{-\mathbf{3}} = -2r(\tau)\gamma^{\bar{\mathbf{3}}}\eta, \\ &\tilde{\psi}_{-\bar{\mathbf{1}}} = s(\tau)\gamma^{\mathbf{1}}\eta, \qquad \tilde{\psi}_{-\bar{\mathbf{2}}} = -s(\tau)\gamma^{\mathbf{2}}\eta, \qquad \tilde{\psi}_{-\bar{\mathbf{3}}} = t(\tau)\gamma^{\mathbf{3}}\eta. \end{aligned} \tag{4.16}$$

One could also add to (4.16) a solution of the homogeneous equation, similar to the $z(\tau)$ dependence of (4.11) that decouples from the system and should be set to zero anyway by imposing regularity at the origin and normalizability. Inserting (4.16) into (3.32) yields three linear first order O.D.E.s in the three unknown functions r, s, t that can be further simplified into two decoupled O.D.E.s (one of first order and the other of second order) for r and s:

$$r'(\tau) + \frac{2\sinh^2\tau}{\cosh\tau\sinh\tau - \tau}r(\tau) = \frac{1}{2}\partial_\tau e^{u(\tau)}, \tag{4.17}$$

[5] We can actually write the solution above in closed form, as $\tilde{\psi}_{+i} = -2iB_{ij}\gamma^j\eta^*$, where B_2 is the 2-form potential of the imaginary part of G_3, also given in [24]. Note that B_2 is a $(1,1)$ primitive form which satisfies $d *_6 B_2 = 0$, conditions which are necessary for consistency with (3.26).

and:

$$s''(\tau) + 4\coth\tau s'(\tau) + 3s(\tau) = -\frac{1}{2}\frac{\partial_\tau e^{u(\tau)}}{\sinh\tau}, \tag{4.18}$$

where we have used (4.15). Furthermore, t is expressed in terms of s:

$$t(\tau) = (s(\tau)\sinh\tau)'. \tag{4.19}$$

The reason why the equations for $s(\tau)$ and $r(\tau)$ decouple is that one can solve separately the equations for the holomorphic and antiholomorphic components.

After some manipulations it turns out that they can both be solved in terms of simple integrals, much like the warp factor:

$$\begin{aligned} r(\tau) &= \frac{1}{2}e^{u(\tau)} - \frac{1}{\sinh\tau\cosh\tau - \tau}\int_0^\tau d\tau' e^{u(\tau')}\sinh^2\tau' \\ &= -M^2\frac{1}{\sinh\tau\cosh\tau - \tau}\int_0^\tau d\tau' K^4(\tau')\sinh\tau'(\tau'\cosh\tau' - \sinh\tau'), \\ s(\tau) &= -\frac{1}{2\sinh^3\tau}\int_0^\tau d\tau' e^{u(\tau')}\sinh^2\tau', \\ &= \frac{1}{4}K^3(\tau)(2r(\tau) - e^{u(\tau)}), \end{aligned} \tag{4.20}$$

where all the integration constants have been fixed so that the solution is regular at the origin and normalizable. Note that $s(\tau)$ is asymptotically subleading, so that the components of the solution depending explicitly on it vanish faster as the boundary is approached.

At last, we conclude this derivation by finding the expression for χ_- from (3.35). Inserting the expression for $\tilde{\psi}^*_{-k}$ in the r.h.s. we find that the source is pointing along $\gamma^{\mathbf{1}}\gamma^{\mathbf{2}}\gamma^{\mathbf{3}}\eta^*$. Perhaps the most convenient way to write the source is in terms of the function $s(\tau)$ in (4.20) and its derivative:

$$\gamma^i D_i\tilde{\chi}_- = \frac{\sqrt{3}M}{4\sinh\tau}(\tau s(\tau) + (\tau\coth\tau - 1)s'(\tau))\gamma^{\mathbf{1}}\gamma^{\mathbf{2}}\gamma^{\mathbf{3}}\eta^*. \tag{4.21}$$

(Notice that $\gamma^{\mathbf{1}}\gamma^{\mathbf{2}}\gamma^{\mathbf{3}}\eta^* \propto \eta$.) This suggests the ansatz

$$\tilde{\chi}_- = w(\tau)\gamma^{\mathbf{1}}\gamma^{\mathbf{2}}\eta^* \tag{4.22}$$

and in fact, (4.21) yields:

$$w(\tau) = \frac{M}{2}\frac{\tau\coth\tau - 1}{K(\tau)\sinh\tau}s(\tau), \tag{4.23}$$

where, once again, the integration constant has been fixed by the requirement that the solution be regular at the origin. One can check that χ_- is also normalizable.

This completes the finding of the zero momentum massless fermionic mode. Having found the explicit solution it is very easy to check that condition (1.2) is obeyed (in the sense that the r.h.s. of (2.6) vanishes) due to the simple expression for the dilatino.

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5. Conclusions

In this paper we began a systematic study of the fermionic equations of motion of IIB supergravity in the context of the gauge/gravity correspondence with emphasis to the search for bulk zero modes dual to massless fermions in the gauge theory. We stressed that among all such fermions, the one associated with SUSY breaking (if it occurs) should be singled out by looking at the gravitino fluctuation, most likely through the contribution to its supercovariant field strength. Other fermionic massless modes, such as the KS "axino" do not contribute to this quantity. It is interesting to note that the vanishing of the r.h.s. of (2.6), at least for our ansatz, is closely related to the conditions for SUSY preservation, namely $G_{ijk}\gamma^{ijk}\eta = 0$. This is no longer vanishing even in the mildest way to break SUSY, i.e. by the presence of a $(0,3)$ piece for G_3.

Another way to distinguish a generic massless fermion from the goldstino is by looking at how it transforms under the global symmetries of the problem. For instance, the bosonic zero modes found in [26, 27] are odd under the Z_2 symmetry exchanging the two S^2 spheres of the deformed conifold and the same symmetry should act non-trivially on its fermionic superpartner.

Let us briefly recall the origin of this symmetry. The exchange of the two spheres is implemented by the exchange of the pairs of coordinate (θ_1, ϕ_1) and (θ_2, ϕ_2) in the solution. Trivially, the sechsbein e^5 and e^6 in (4.1) are invariant under the exchange but the remaining four transform in a complicated way. One can construct however various combinations that transform in a simple way:

$$(e^1)^2 + (e^2)^2 \quad \text{and} \quad (e^3)^2 + (e^4)^2 \tag{5.1}$$

that are even under the exchange, and

$$e^1 \wedge e^2, \quad e^3 \wedge e^4 \quad \text{and} \quad e^1 \wedge e^3 + e^2 \wedge e^4 \tag{5.2}$$

that are odd. Hence, of the bosonic fields, the (constant) dilaton, the metric and the five-form are even while the three form G_3 is odd. Let us also recall that the bosonic zero mode $a(x)$ constructed in [26] enters as $\delta G_3 = *_4 da + \dots$ and thus it must be odd under the symmetry so as to preserve the overall parity of G_3.

Let us now look at the fermionic solution presented in section 4. The transformation properties of the fields ψ_{+i} and ψ_{-i} are somewhat complicated by the fact that they carry an internal index but we don't really need them for the argument — it is quite enough to look at $\chi_\pm$ and $\lambda_\pm$.

The claim is that χ_+ and λ_- are even while χ_- is odd (λ_+ is zero). To see this, notice that the covariantly constant spinors η and η^* are both even because the six-dimensional chirality is unchanged by the symmetry (one is exchanging two pairs of indices). Thus χ_+ and λ_- given in (4.8) and (4.14) are even.[6] On the other hand, expanding the solution (4.22) for χ_- in terms of the gamma matrices in the real basis, one gets a spinor proportional to

$$\left(\gamma^1\gamma^3 + \gamma^2\gamma^4 + i(\gamma^1\gamma^2 - \gamma^3\gamma^4)\right)\eta^* \tag{5.3}$$

[6] The quantities with a tilde only differ by powers of the warp factor and have thus the same transformation properties.

that transforms as the combinations of sechsbein in (5.2) and it is thus odd. To compensate for that in the expression for Ψ_μ we must let the zero mode $\epsilon_\pm \to -\gamma_{\chi 4}\epsilon_\pm$, thus showing that it transforms non-trivially under the Z_2 symmetry.

The second point briefly mentioned in the introduction that we would like to discuss is the possibility that the presence of the warp factor might allow for normalizable zero modes without requiring a Kähler structure and thus SUSY.

There are two different types of boundary conditions that should be considered. Let us begin with the standard one. Assume that one is looking at an internal manifold whose metric is asymptotically that of a cone:

$$d\hat{s}^2 \approx dr^2 + r^2 d\Sigma^2, \tag{5.4}$$

for some five-dimensional Einstein (but not necessarily Sasaki) manifold with metric $d\Sigma^2$. The existence of a covariantly constant spinor would of course imply a Kähler structure for the manifold, but the original condition (the Dirac equation (3.24)) necessary for the zero mode is weaker on a non-compact manifold. The two conditions are equivalent by the standard argument of integration by part only for spinors $\tilde{\chi}_+$ vanishing at infinity faster that r^{-2}. (The covariantly constant spinor η is an exception because it is completely independent on r.)

Would the existence of a spinor solving the Dirac equation (3.24) but decaying more slowly than r^{-2} still allow for a massless mode on the boundary? For this we must look at the other boundary condition inferred from the AdS/CFT correspondence in [43]. Namely, we must ensure that, for instance:

$$\left(\sqrt{G}\bar{\chi}_+\chi_+\right)\Big|_{\rm bdry} < \infty, \tag{5.5}$$

where G is the determinant of the induced metric at the boundary. This is one of the conditions that has been used throughout section 4 to check for normalizability. Inserting the appropriate powers of the warp factor $e^u \approx r^{-4}$, one sees that (5.5) requires $\tilde{\chi}_+$ to scale like r^α with $\alpha < 1/2$. Thus, the possibility of having a normalizable zero mode without a Kähler structure is left open.

Whether gravity duals to theories with a stable non-supersymmetric vacuum exist is still an open and interesting question. It seems that one would need the background to be dual to a chiral gauge theory with no classical flat directions (see the arguments and caveats in [51]). The cascading theories considered until now are not of this kind, and we do not expect a smooth gravity dual for a theory with no stable vacuum. Presumably, one will have to turn to a more general ansatz, but we hope that a similar analysis to the one performed in this paper can be helpful in this endeavor.

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A. Some useful formulas

We collect some useful formulas that have been used extensively in the derivation of the equations in the text.

Let us begin with the spin connection. In the flat basis, the non-zero components of the spin connection $\omega^{ab,c} = -\omega^{ba,c}$ are

$$\begin{aligned}
\omega^{12,1} &= \omega^{34,1} = \frac{\cos\theta_1}{2A(\tau)\sin\theta_1} \\
\omega^{16,1} &= -\omega^{45,1} = \omega^{26,2} = \omega^{35,2} = f_1(\tau) \\
\omega^{12,3} &= \omega^{34,3} = \frac{\cos\theta_1}{2B(\tau)\sin\theta_1} \\
\omega^{36,3} &= -\omega^{25,3} = \omega^{46,4} = \omega^{15,4} = f_2(\tau) \\
\omega^{12,5} &= \omega^{34,5} = \frac{1}{2C(\tau)} \\
\omega^{23,5} &= -\omega^{14,5} = \frac{1}{2}\omega^{56,5} = f_3(\tau)
\end{aligned} \tag{A.1}$$

where in defining the functions f_1, f_2, f_3 we have taken into account the following identities:

$$\begin{aligned}
f_1(\tau) &= \frac{A'(\tau)}{A(\tau)C(\tau)} = \frac{-A^2(\tau)+B^2(\tau)+C^2(\tau)}{4A(\tau)B(\tau)C(\tau)} \\
f_2(\tau) &= \frac{B'(\tau)}{B(\tau)C(\tau)} = \frac{A^2(\tau)-B^2(\tau)+C^2(\tau)}{4A(\tau)B(\tau)C(\tau)} \\
f_3(\tau) &= \frac{C'(\tau)}{2C^2(\tau)} = \frac{A^2(\tau)+B^2(\tau)-C^2(\tau)}{4A(\tau)B(\tau)C(\tau)}.
\end{aligned} \tag{A.2}$$

Other useful formulas are those involving the self-dual forms G_3 and F_5:

$$\begin{aligned}
\gamma^{ijk}G_{ijk} &= \gamma^{ijk}G_{ijk}\frac{1+\gamma_{\chi 6}}{2} \\
\gamma^m\gamma^{ijk}G_{ijk} &= 6\gamma_{ij}G^{mij}\frac{1+\gamma_{\chi 6}}{2} \\
\gamma^{ijk}\gamma^m G_{ijk} &= 6\gamma_{ij}G^{mij}\frac{1-\gamma_{\chi 6}}{2},
\end{aligned} \tag{A.3}$$

and similarly for the complex conjugates (recall that γ_i and $\gamma_{\chi 6}$ are all imaginary in order for the ten dimensional matrices to be real). The terms containing the five-form on the other hand, can be simplified with the help of:

$$\frac{i}{240}\Gamma^{MNPQR}F_{MNPQR} = \frac{1}{4}\partial_m u \Gamma^m \Gamma_{\chi 6}\frac{1-\Gamma_{\chi 10}}{2}, \tag{A.4}$$

where $\Gamma_{\chi 10} = \Gamma^0 \dots \Gamma^9$ as in the text and $\Gamma_{\chi 6} = -i\Gamma^4 \dots \Gamma^9 = \mathbf{1} \otimes \gamma_{\chi 6}$ all with flat indices. (See also e.g. [52, 53].)

Lastly, recall that, since D_i represents the covariant derivative on the internal manifold, it no longer commutes with Γ_μ and we have instead:

$$D_i \Gamma_\mu = \Gamma_\mu D_i - \frac{1}{4}\partial_i u \Gamma_\mu. \tag{A.5}$$

References

[1] O. Aharony, *The non-AdS/non-CFT correspondence, or three different paths to QCD*, hep-th/0212193.

[2] M. Bertolini, *Four lectures on the gauge-gravity correspondence*, *Int. J. Mod. Phys.* **A 18** (2003) 5647 [hep-th/0303160].

[3] M.J. Strassler, *The duality cascade*, hep-th/0505153.

[4] D. Berenstein, C.P. Herzog, P. Ouyang and S. Pinansky, *Supersymmetry breaking from a Calabi-Yau singularity*, *JHEP* **09** (2005) 084 [hep-th/0505029].

[5] S. Franco, A. Hanany, F. Saad and A.M. Uranga, *Fractional branes and dynamical supersymmetry breaking*, *JHEP* **01** (2006) 011 [hep-th/0505040].

[6] M. Bertolini, F. Bigazzi and A.L. Cotrone, *Supersymmetry breaking at the end of a cascade of Seiberg dualities*, *Phys. Rev.* **D 72** (2005) 061902 [hep-th/0505055].

[7] D. Martelli and J. Sparks, *Toric geometry, Sasaki-Einstein manifolds and a new infinite class of AdS/CFT duals*, *Commun. Math. Phys.* **262** (2006) 51 [hep-th/0411238].

[8] M. Bertolini, F. Bigazzi and A.L. Cotrone, *New checks and subtleties for AdS/CFT and a-maximization*, *JHEP* **12** (2004) 024 [hep-th/0411249].

[9] S. Benvenuti, S. Franco, A. Hanany, D. Martelli and J. Sparks, *An infinite family of superconformal quiver gauge theories with Sasaki-Einstein duals*, *JHEP* **06** (2005) 064 [hep-th/0411264].

[10] J.P. Gauntlett, D. Martelli, J. Sparks and D. Waldram, *Sasaki-Einstein metrics on $S^2 \times S^3$*, *Adv. Theor. Math. Phys.* **8** (2004) 711 [hep-th/0403002].

[11] J.P. Gauntlett, D. Martelli, J.F. Sparks and D. Waldram, *A new infinite class of Sasaki-Einstein manifolds*, *Adv. Theor. Math. Phys.* **8** (2006) 987 [hep-th/0403038].

[12] K. Intriligator and N. Seiberg, *The runaway quiver*, *JHEP* **02** (2006) 031 [hep-th/0512347].

[13] I. Affleck, M. Dine and N. Seiberg, *Dynamical supersymmetry breaking in supersymmetric QCD*, *Nucl. Phys.* **B 241** (1984) 493.

[14] C.P. Herzog, Q.J. Ejaz and I.R. Klebanov, *Cascading RG flows from new Sasaki-Einstein manifolds*, *JHEP* **02** (2005) 009 [hep-th/0412193].

[15] B.A. Burrington, J.T. Liu, M. Mahato and L.A. Pando Zayas, *Towards supergravity duals of chiral symmetry breaking in sasaki-einstein cascading quiver theories*, *JHEP* **07** (2005) 019 [hep-th/0504155].

[16] S.S. Pal, *A new Ricci flat geometry*, *Phys. Lett.* **B 614** (2005) 201 [hep-th/0501012].

[17] K. Sfetsos and D. Zoakos, *Supersymmetric solutions based on $Y^{p,q}$ and $L^{p,q,r}$*, *Phys. Lett.* **B 625** (2005) 135 [hep-th/0507169].

[18] M. Berg, M. Haack and W. Muck, *Bulk dynamics in confining gauge theories*, *Nucl. Phys.* **B 736** (2006) 82 [hep-th/0507285].

[19] D.N. Page and C.N. Pope, *Inhomogeneous Einstein metrics on complex line bundles*, *Class. and Quant. Grav.* **4** (1987) 213.

[20] E. Witten, *Dynamical breaking of supersymmetry*, *Nucl. Phys.* **B 188** (1981) 513.

[21] P. Fayet and J. Iliopoulos, *Spontaneously broken supergauge symmetries and goldstone spinors*, *Phys. Lett.* **B 51** (1974) 461.

[22] L. O'Raifeartaigh, *Spontaneous symmetry breaking for chiral scalar superfields*, *Nucl. Phys.* **B 96** (1975) 331.

[23] A. Salam and J.A. Strathdee, *On Goldstone fermions*, *Phys. Lett.* **B 49** (1974) 465.

[24] I.R. Klebanov and M.J. Strassler, *Supergravity and a confining gauge theory: duality cascades and χSB-resolution of naked singularities*, *JHEP* **08** (2000) 052 [hep-th/0007191].

[25] O. Aharony, *A note on the holographic interpretation of string theory backgrounds with varying flux*, *JHEP* **03** (2001) 012 [hep-th/0101013].

[26] S.S. Gubser, C.P. Herzog and I.R. Klebanov, *Symmetry breaking and axionic strings in the warped deformed conifold*, *JHEP* **09** (2004) 036 [hep-th/0405282].

[27] S.S. Gubser, C.P. Herzog and I.R. Klebanov, *Variations on the warped deformed conifold*, *Comptes Rendus Physique* **5** (2004) 1031 [hep-th/0409186].

[28] M. Grana, *Flux compactifications in string theory: a comprehensive review*, *Phys. Rept.* **423** (2006) 91 [hep-th/0509003].

[29] J.H. Schwarz, *Covariant field equations of chiral $N = 2$ $D = 10$ supergravity*, *Nucl. Phys.* **B 226** (1983) 269.

[30] J.H. Schwarz and P.C. West, *Symmetries and transformations of chiral $N = 2$ $D = 10$ supergravity*, *Phys. Lett.* **B 126** (1983) 301.

[31] P.S. Howe, K.S. Stelle and P.K. Townsend, *Miraculous ultraviolet cancellations in supersymmetry made manifest*, *Nucl. Phys.* **B 236** (1984) 125.

[32] A.R. Kavalov and R.L. Mkrtchian, *Lagrangian of the selfduality equation and $D = 10$, $N = 2b$ supergravity*, *Sov. J. Nucl. Phys.* **46** (1987) 728.

[33] L. Castellani and I. Pesando, *The complete superspace action of chiral $D = 10$, $N = 2$ supergravity*, *Int. J. Mod. Phys.* **A 8** (1993) 1125.

[34] M. Grana and J. Polchinski, *Supersymmetric three-form flux perturbations on AdS_5*, *Phys. Rev.* **D 63** (2001) 026001 [hep-th/0009211].

[35] A. Butti, M. Grana, R. Minasian, M. Petrini and A. Zaffaroni, *The baryonic branch of Klebanov-Strassler solution: a supersymmetric family of* SU(3) *structure backgrounds*, *JHEP* **03** (2005) 069 [hep-th/0412187].

[36] M. Grana, R. Minasian, M. Petrini and A. Tomasiello, *Supersymmetric backgrounds from generalized Calabi-Yau manifolds*, *JHEP* **08** (2004) 046 [hep-th/0406137].

[37] A. Dymarsky, I.R. Klebanov and N. Seiberg, *On the moduli space of the cascading* $\mathrm{SU}(M+p)\times\mathrm{SU}(p)$ *gauge theory*, *JHEP* **01** (2006) 155 [hep-th/0511254].

[38] S. Kuperstein and J. Sonnenschein, *Analytic non-supersymmetric background dual of a confining gauge theory and the corresponding plane wave theory of hadrons*, *JHEP* **02** (2004) 015 [hep-th/0309011].

[39] P. Candelas and X.C. de la Ossa, *Comments on conifolds*, *Nucl. Phys.* **B 342** (1990) 246.

[40] R. Minasian and D. Tsimpis, *On the geometry of non-trivially embedded branes*, *Nucl. Phys.* **B 572** (2000) 499 [hep-th/9911042].

[41] K. Ohta and T. Yokono, *Deformation of conifold and intersecting branes*, *JHEP* **02** (2000) 023 [hep-th/9912266].

[42] D. Arean, D.E. Crooks and A.V. Ramallo, *Supersymmetric probes on the conifold*, *JHEP* **11** (2004) 035 [hep-th/0408210].

[43] M. Henningson and K. Sfetsos, *Spinors and the AdS/CFT correspondence*, *Phys. Lett.* **B 431** (1998) 63 [hep-th/9803251].

[44] W. Muck and K.S. Viswanathan, *Conformal field theory correlators from classical field theory on anti-de Sitter space, II. Vector and spinor fields*, *Phys. Rev.* **D 58** (1998) 106006 [hep-th/9805145].

[45] G.E. Arutyunov and S.A. Frolov, *On the origin of supergravity boundary terms in the AdS/CFT correspondence*, *Nucl. Phys.* **B 544** (1999) 576 [hep-th/9806216].

[46] M. Henneaux, *Boundary terms in the AdS/CFT correspondence for spinor fields*, hep-th/9902137.

[47] S. Corley, *The massless gravitino and the AdS/CFT correspondence*, *Phys. Rev.* **D 59** (1999) 086003 [hep-th/9808184].

[48] A. Volovich, *Rarita-Schwinger field in the AdS/CFT correspondence*, *JHEP* **09** (1998) 022 [hep-th/9809009].

[49] R.C. Rashkov, *Note on the boundary terms in AdS/CFT correspondence for Rarita-Schwinger field*, *Mod. Phys. Lett.* **A 14** (1999) 1783 [hep-th/9904098].

[50] P. Matlock and K.S. Viswanathan, *The AdS/CFT correspondence for the massive Rarita-Schwinger field*, *Phys. Rev.* **D 61** (2000) 026002 [hep-th/9906077].

[51] I. Affleck, M. Dine and N. Seiberg, *Dynamical supersymmetry breaking in four-dimensions and its phenomenological implications*, *Nucl. Phys.* **B 256** (1985) 557.

[52] A. Kehagias, *New type-IIB vacua and their F-theory interpretation*, *Phys. Lett.* **B 435** (1998) 337 [hep-th/9805131].

[53] S.S. Gubser, *Supersymmetry and F-theory realization of the deformed conifold with three-form flux*, hep-th/0010010.